

# Analysis of Incomplete Non-Normal Longitudinal Lipid Data

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# Outline

- Standard methods and Existing issues
- Case study (Merck trial)
- Proposed robust method
- Simulation results
- Conclusions

## Standard Methods and Existing issues

- In lipid study, we often encounter
  - Non-normal parameters, such as Triglycerides, Lipoprotein (a) etc.
  - Outliers
  - Drop-out: not significant but still very common (10-15%)

# Standard Methods and Existing issues

- Current standard Methods
  - LOCF (Last value carried forward) + Rank based analysis
    - Do not utilize the longitudinal nature of the data;
    - Biased estimation
  - Standard Longitudinal Data Analysis Method: Restricted Maximum Likelihood (REML) (Laird & Ware, 1982)
    - Key Assumption: Multivariate normality of the residual vector

# Standard Methods and Existing issues

- Alternative Methods
  - Generalized Estimating Equations (GEE) (Liang and Zeger 1986)  
Alternative to REML if normality is in question  
Missing data (if any) are missing completely at random (MCAR)
  - Weighted GEE (WGEE)  
Relaxes MCAR assumption to missing at random (MAR) by adding weights to GEE

## Case Study

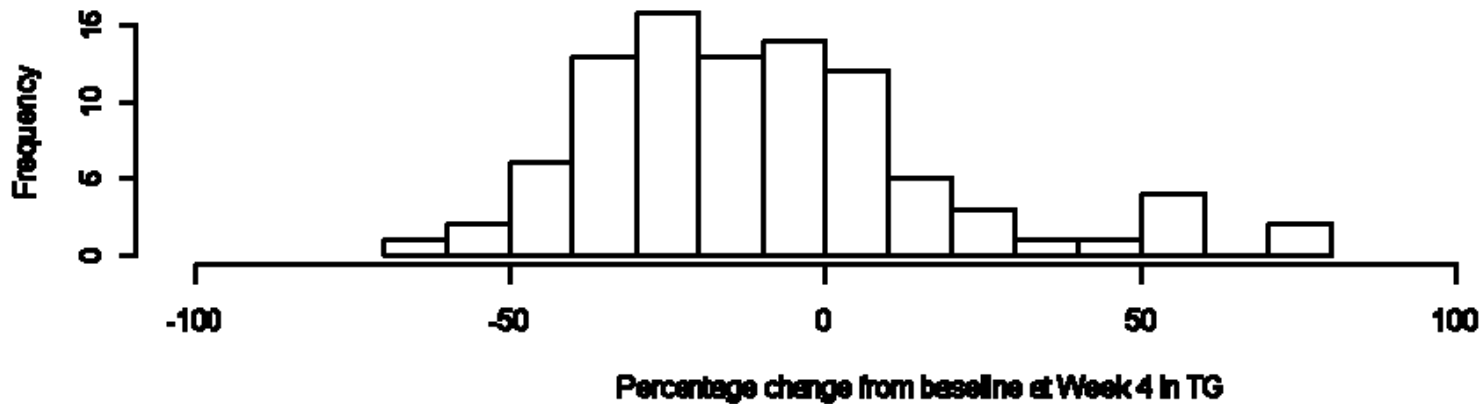
Drug (A) vs. placebo (B), 3:2 randomization

- Patients with Dyslipidemia
- Triglycerides (TG) measured at  $t = 0$  (baseline), 2 and 4 weeks
- $Y_{tjk}$  = % change from baseline (post-pre) for time  $t$ , trt  $j$ , subject  $k$ , and let  $\mu_{tj} = E(Y_{tjk})$
- **Objectives:**
  - Estimation of treatment difference at time T  
 $\delta = \mu_{TA} - \mu_{TB}$  (T = 4 weeks)
  - Hypothesis testing  
 $H_0: \delta = 0$  vs.  $H_1: \delta \neq 0$

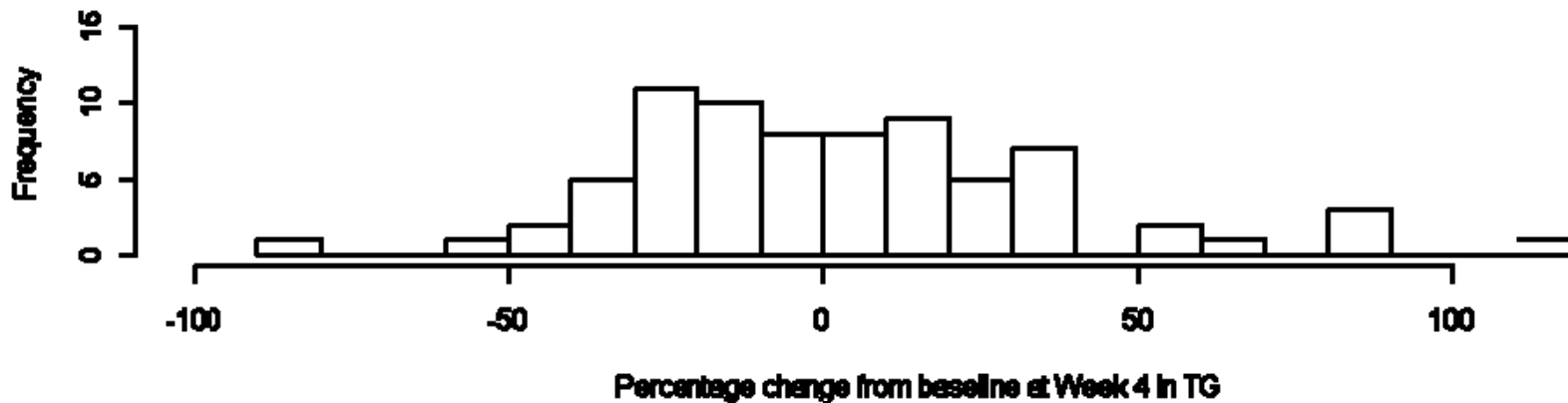
## Case Study (continued)

### % Change from baseline at Week 4 in TG

**Treatment**



**Placebo**



The data has significant outliers.

Dropouts: Treatment  $19/112=17\%$ ; Placebo  $1/75=1\%$

## Case Study (continued)

### Standard LDA Methods: Results

METHOD	Estimate of $\delta$	Standard Error	95 % CI for $\delta$	2-tailed p-value
LOCF+Rank	-7.47	4.08	(-15.47, 0.52)	0.068
REML	-8.90	4.45	(-17.68, -0.12)	0.047
GEE	-8.80	4.52	(-17.66, 0.06)	0.052
WGEE	-8.83	4.57	(-17.80, 0.13)	0.053

LOCF+ Rank = Last value carry forward. Use turkey normalized ranks.

REML = mixed effects model with restricted maximum likelihood estimation (Laird & Ware, 1982)

GEE = generalized estimating equations (Liang & Zeger, 1986)

WGEE = weighted GEE (Robins, Rotnitzsky, & Zhao, 1995)

Covariance structures: UN for REML and AR(1) for GEE/WGEE

SAS implementation: PROC MIXED for REML and PROC GENMOD for GEE/WGEE

**Is there a statistically significant drug effect?**



## Proposed Robust Method

- We propose a simple "adaptive" robust procedure: Mehrotra, Li, Liu, and Lu (submitted)  
Test for **normality** at .001 level  
Normality not rejected  $\Rightarrow$  use REML  
Normality rejected  $\Rightarrow$  use **MI  $\rightarrow$  RR**

### **MI $\rightarrow$ RR**

- Create  $C$  (=20) complete datasets using **multiple imputation** of missing values **{SAS: PROC MI}**
- Analyze each imputed dataset via **robust regression** for endpoint of interest (e.g.,  $Y = \% \text{ change from baseline at week 4}$ ) **{SAS: PROC ROBUSTREG}**
- Combine the  $C$  results for estimation/inference using a **plug-in formula** (Rubin, 1987)

## Proposed Robust Method (continued)

### Test for Normality

- Test is based on the mixed model residuals scaled by the inverse Cholesky root of the marginal variance-covariance matrix (**VCIRY** option).

- PROC MIXED DATA=a;  
CLASS trt time sub;  
MODEL resp=x trt time trt\*time/OUTPM=b RESIDUAL **VCIRY**;  
REPEATED time/TYPE=UN SUBJECT=sub(trt);  
RUN;

```
PROC UNIVARIATE DATA=b NORMAL;  
VAR scaledresid;  
OUTPUT OUT=mvntest PROBN=swprob; * Shapiro-Wilks (1965) test;  
RUN;
```

## Proposed Robust Method (continued)

# Robust Regression for a Single Imputed Dataset

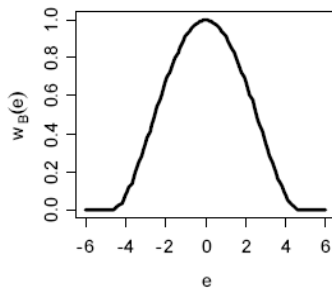
- Regression model:

$$Y = \beta_0 + \beta_1 Z + (\text{covariate terms}) + \varepsilon$$

$$E(\varepsilon) = 0, \quad V(\varepsilon) = \sigma^2, \quad Z = \begin{cases} 1 & \text{trt A} \\ 0 & \text{trt B} \end{cases}$$

Null:  $\beta_1 = 0$  Alternative:  $\beta_1 \neq 0$   $(\beta_1 \equiv \delta)$

- $\tilde{\beta}_1 = \mathbf{M}$ -estimate of  $\beta_1$  with variance  $V(\tilde{\beta}_1)$   
(obtained via SAS PROC ROBUSTREG default)  
M-estimation = weighted least squares (Huber 1973)



Default weight function for M-estimation;  
**outliers** get automatically down-weighted!

## Proposed Robust Method (continued)

### Combining Results from Multiple Imputed Datasets

- Let  $\tilde{\beta}_{1i} (\equiv \tilde{\delta}_i)$  be the **M-estimate** of  $\beta_1 (\equiv \delta)$  for  $i$ -th of  $C$  imputed (i.e., completed) datasets.

$$Z_{MI \rightarrow RR} = \frac{\frac{1}{C} \sum_i \tilde{\delta}_i}{\sqrt{\frac{1}{C} \sum_i V(\tilde{\delta}_i) + \left(\frac{C+1}{C}\right) \left(\frac{1}{C-1}\right) \sum_i (\tilde{\delta}_i - \tilde{\delta})^2}} \equiv \frac{\tilde{\delta}}{\sqrt{V(\tilde{\delta})}}$$

Rubin (1987) variance is used above. **Robins & Wang [R-W] (2000) variance is "more accurate" but very complicated!**

- p-value =  $2 \times P\{Z > |Z_{MI \rightarrow RR}|\}$
- Confidence interval =  $\tilde{\delta} \pm Z_{\alpha/2} \sqrt{V(\tilde{\delta})}$

# Case Study Revisited

## Analysis of % change from baseline in TG

METHOD	Estimate of $\delta$	Standard Error	95 % CI For $\delta$	2-tailed p-value
LOCF+Rank	-7.47	4.08	(-15.47, 0.52)	0.068
REML	-8.90	4.45	(-17.68, -0.12)	0.047
GEE	-8.80	4.52	(-17.66, 0.06)	0.052
WGEE	-8.83	4.57	(-17.80, 0.13)	0.053
MI $\rightarrow$ RR	-11.1	4.60	(-20.12, -2.08)	0.022
Adaptive*	-11.1	4.60	(-20.12, -2.08)	0.022

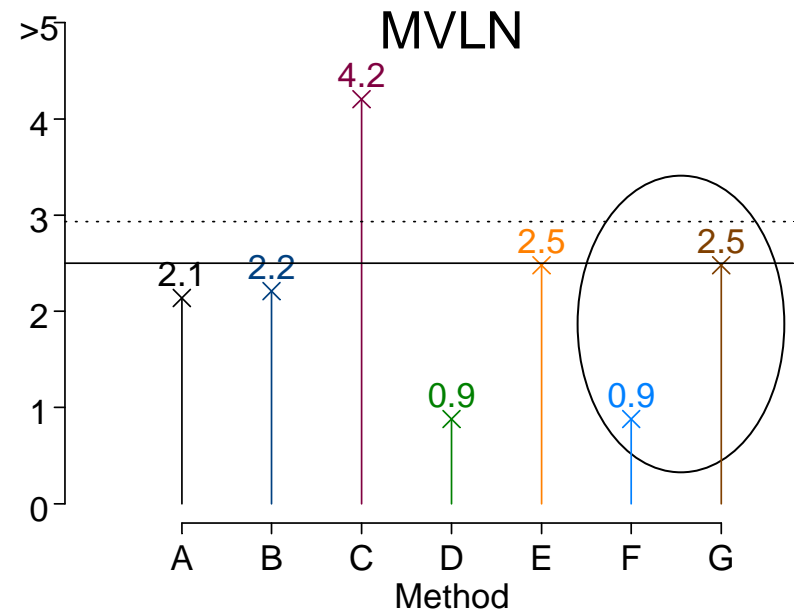
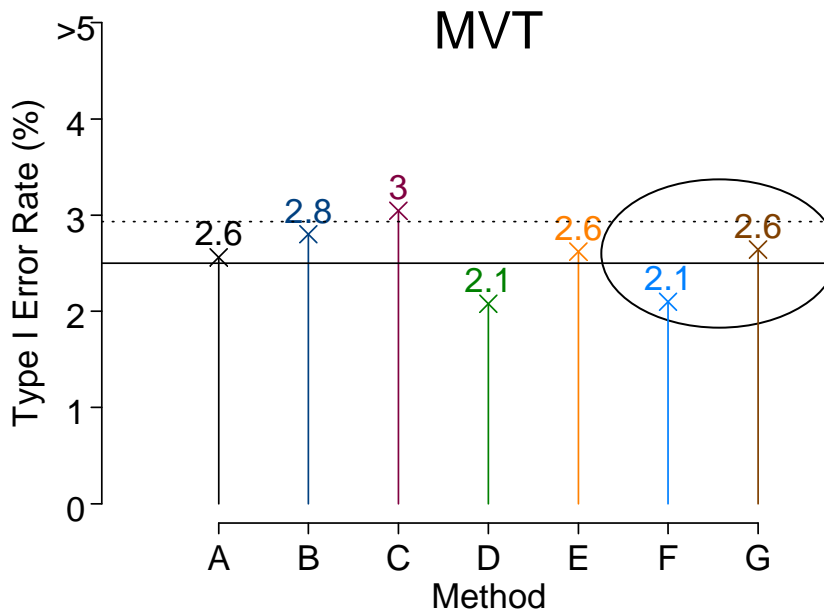
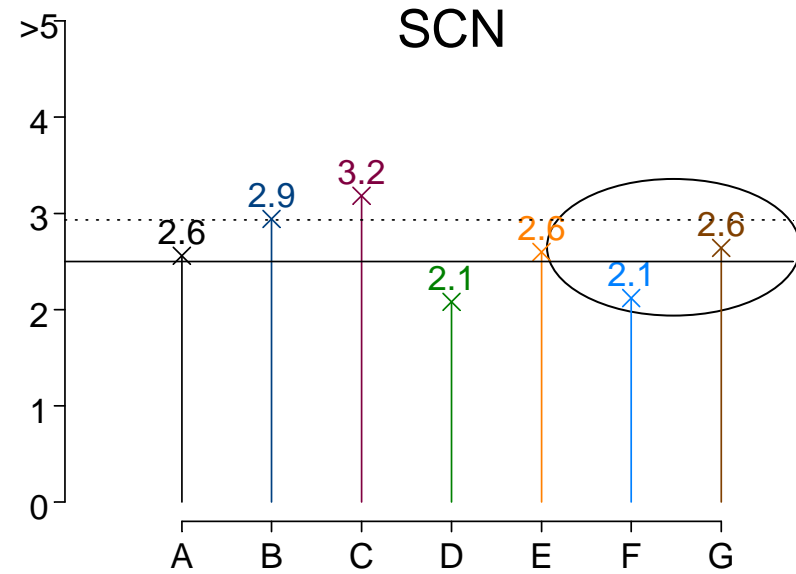
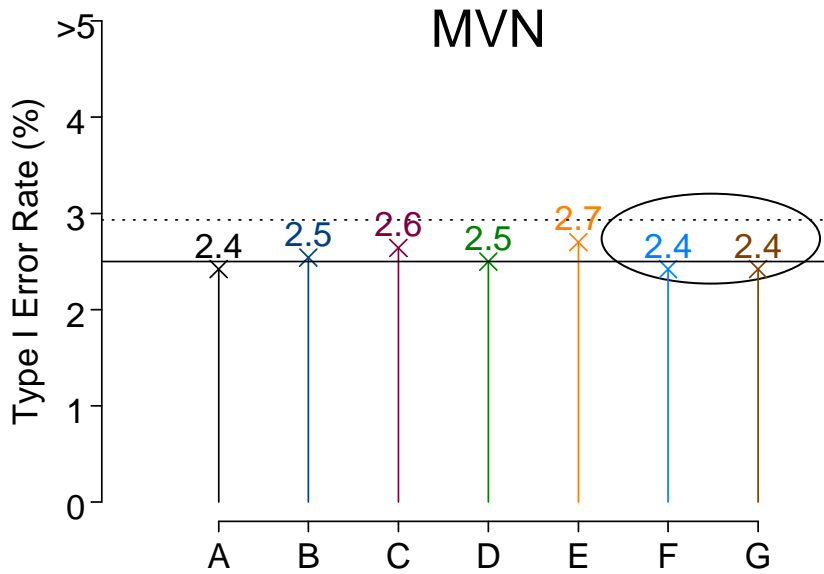
\* MI  $\rightarrow$  RR is chosen because normality is rejected at the .001 level

This is not an isolated example.

**Simulations** confirm that the proposed method:

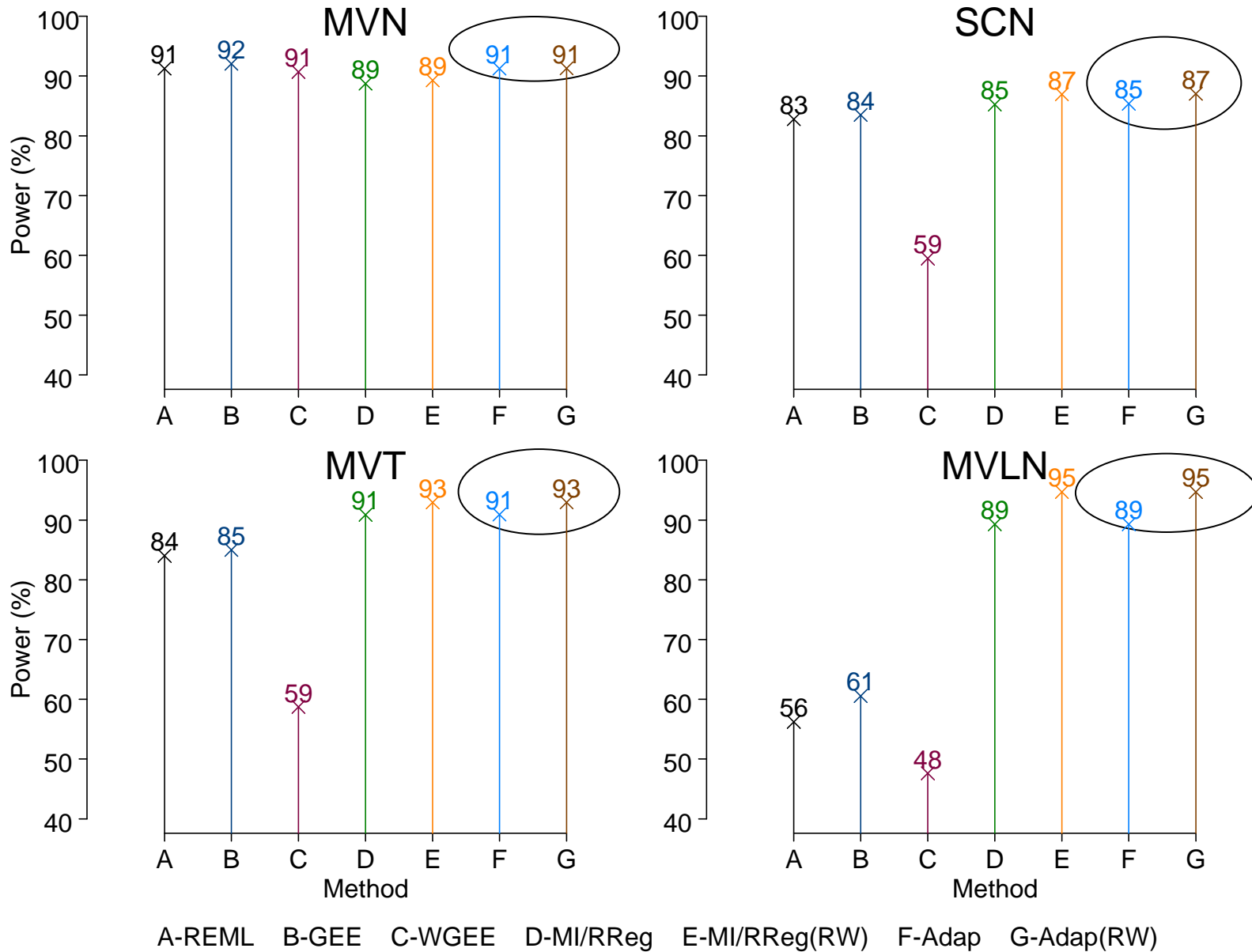
- Is as powerful as REML under normality.
- Is much more powerful than REML/GEE/WGEE under non-normality.
- Controls the type I error rate.

# Type I Error Rate ( $\alpha=2.5\%$ ) [80/group]



A-REML B-GEE C-WGEE D-MI/RRreg E-MI/RRreg(RW) F-Adap G-Adap(RW)

# Power (%) [80/group]



## Conclusions

- Traditional analysis method on non-normal lipid data lacks robustness against its **non-normality/outliers**, which can increase the risk of erroneous conclusions.
- **MI → RR** is a robust, flexible, and easy-to-implement alternative to REML, GEE, and WGEE that simultaneously tackles the challenge of missing data and non-normality/outliers.

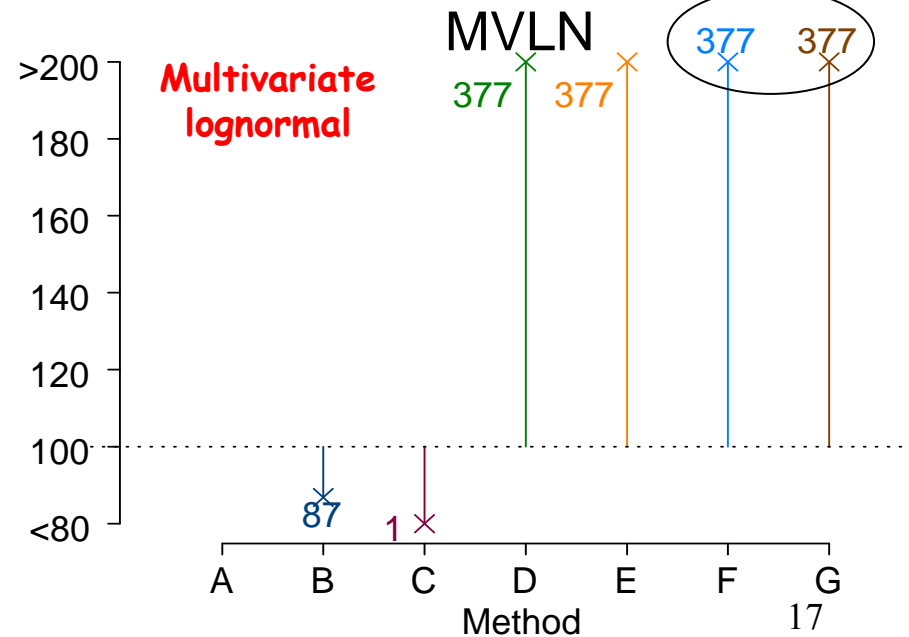
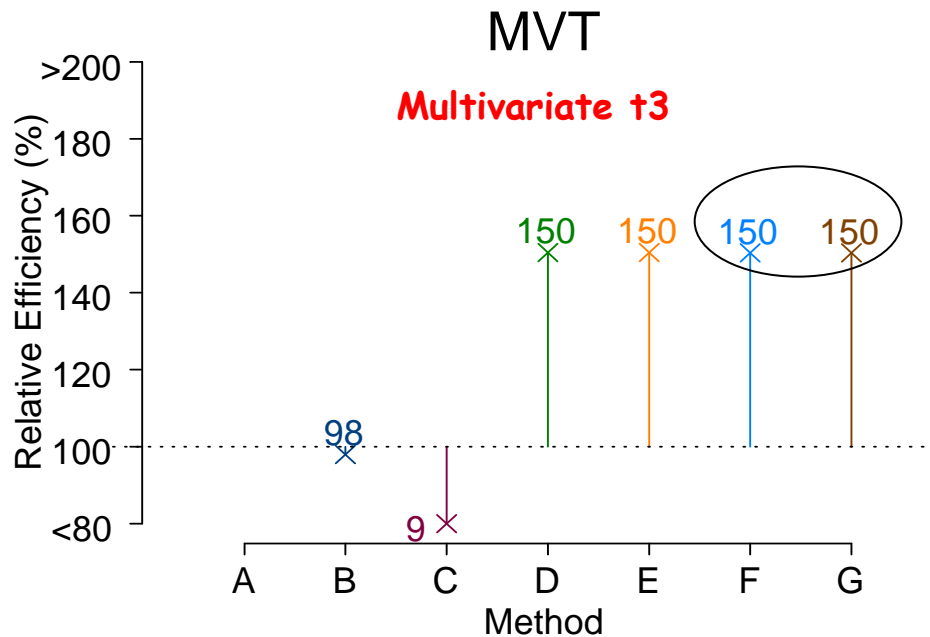
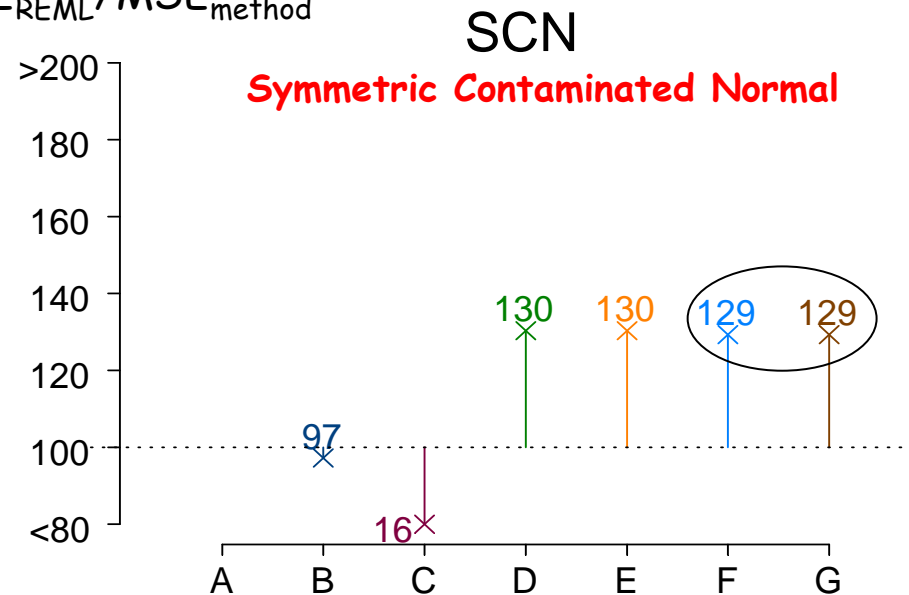
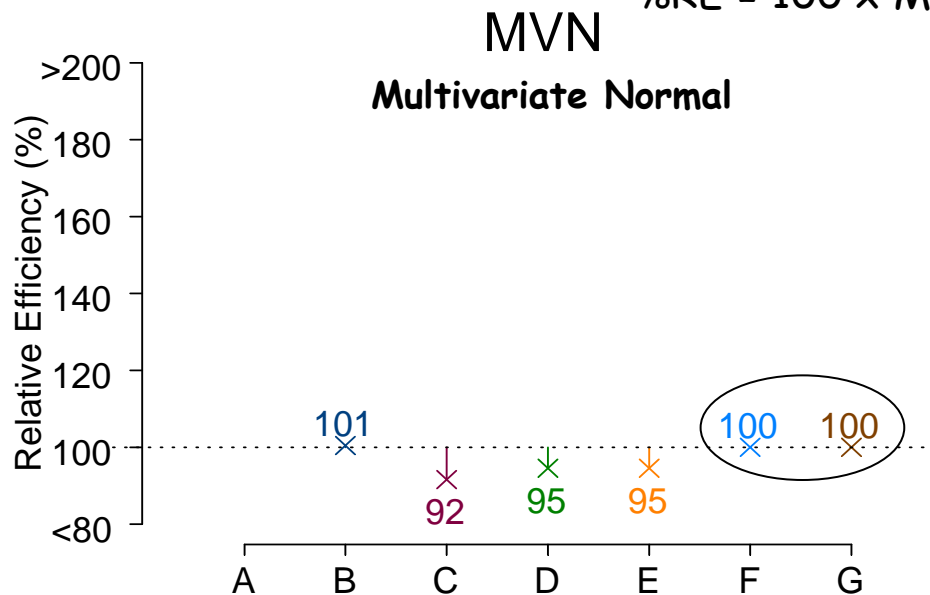
Note: using the Robins & Wang (2000) [RW] variance can improve power compared with the Rubin (1987) variance, but the former is very complicated!

**RECOMMENDATION:** Adaptive method (choosing between REML and MI → RR based on a test of normality) is recommended for routine use.



# % Relative Efficiency vs. REML [80/group]

$$\%RE = 100 \times \text{MSE}_{\text{REML}} / \text{MSE}_{\text{method}}$$



A-REML    B-GEE    C-WGEE    D-MI/RRreg    E-MI/RRreg(RW)    F-Adap    G-Adap(RW)