

Bayesian Dose-Finding in Two Treatment Cycles based on the Joint Utility of Efficacy and Toxicity

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Collaborators

Juhee Lee : Model formulation, programming, heavy lifting - - and she laughs at all my jokes

Peter Mueller: Vast wisdom, Austro-Bayesian perspective

Yuan Ji: Knowledge of dose-finding and mathematics

Phase I-II Dose-Finding in Two Cycles

Goal: Develop a practical phase I-II trial design to adaptively optimize each patient's doses, d_1 and d_2 , in two cycles of therapy, using binary $(Y, Z) = (\text{Toxicity}, \text{Efficacy})$ in each cycle.

Methodology: Base cycle-specific actions on numerical utilities, $U(y, z)$, for $(y, z) = (0,0), (1,0), (0,1),$ or $(1,1)$

1. **Action in each cycle** : Treat with the “optimal” dose, or do not to treat, NT .
2. **Dose-outcome model:** Bayesian hierarchical
3. **Optimization of (a_1, a_2) :** Apply Bellman (1957), using posterior means of a model-based objective function
4. **Safety:** Include additional dose acceptability rules

Doses, Outcomes, and Actions

Dose set $\{1, \dots, m\}$ Action set $\mathcal{A} = \{1, \dots, m\} \cup \{NT\}$

$d_{i,c}$ = dose, $Y_{i,c}$ = I(Toxicity), $Z_{i,c}$ = I(Efficacy) of pat. i in cycle c

$\mathbf{Y}_i = (Y_{i,1}, Y_{i,2})$, $\mathbf{Z}_i = (Z_{i,1}, Z_{i,2})$, $\mathbf{d}_i = (d_{i,1}, d_{i,2})$

$\mathcal{X}_t = \{(\mathbf{Y}_i, \mathbf{Z}_i, \mathbf{d}_i) : i = 1, \dots, n_t\}$ = current data at trial time t

$a_{i,c}$ = action taken for patient i in cycle $c = 1$ or 2

$a_{i,1}$ maps \mathcal{X}_t to \mathcal{A} \Rightarrow Adaptive between patients

$a_{i,2}$ maps [\mathcal{X}_t , cycle 1 data = $(d_{i,1}, Y_{i,1}, Z_{i,1})$] to \mathcal{A}
 \Rightarrow Adaptive both between and within patients

Actions versus Doses

Bellman's Idea: First find a_2^{opt} by considering all possibilities, then work backwards to find a_1^{opt} , assuming that a_2^{opt} will be taken.

Finding $\mathbf{a}^{opt} = (a_1^{opt}, a_2^{opt})$ is not the same thing as optimizing doses separately in each cycle, $\mathbf{d}^{opt} = (d_1^{opt}, d_2^{opt})$.

Example: $(d_1^{opt}, d_2^{opt}) = (3, 2)$, but $\mathbf{a}^{opt} = (3, a_2^{opt})$ with

$$a_2^{opt} = 3 \quad \text{if } (Y_1, Z_1) = (0, 1) = \text{No Tox} + \text{Eff}$$

$$a_2^{opt} = 4 \quad \text{if } (Y_1, Z_1) = (0, 0) = \text{No Tox} + \text{No Eff}$$

$$a_2^{opt} = 1 \quad \text{if } (Y_1, Z_1) = (1, 1) = \text{Tox} + \text{Eff}$$

$$a_2^{opt} = NT \quad \text{if } (Y_1, Z_1) = (1, 0) = \text{Tox} + \text{No Eff}$$

Joint distribution for $[Y_i, Z_i \mid d_i]$

An Ancient Technique: Use 4 continuous normal latent variables to define 4 discrete observed variables. Induces association among the discrete variables, facilitates posterior computation.

Real-valued, cycle-specific latent variables:

$$\xi_i = (\xi_{i,1}, \xi_{i,2}) \text{ for } Y_i \text{ and } \eta_i = (\eta_{i,1}, \eta_{i,2}) \text{ for } Z_i$$

Observables: $Y_{i,c} = \mathbf{I}(\xi_{i,c} > 0)$ and $Z_{i,c} = \mathbf{I}(\eta_{i,c} > 0) \Rightarrow$

$$p(Y_i, Z_i \mid d_i) \text{ is induced by } p(\xi_i, \eta_i \mid d_i)$$

$(\xi_i, \eta_i) \mid d_i \sim$ multiv. normal, means vary with d_i and patient random effects, (u_i, v_i) that induce association

Priors of the Hierarchical Model

Level 1 Priors on the Latent Variables (ξ_i, η_i)

For patient i in cycle c given dose $d_{i,c} = d$,

$$\xi_{i,c} \mid \mathbf{u}_i, \bar{\xi}_{c,d}, \sigma_\xi^2 \sim \text{N}(\bar{\xi}_{c,d} + \mathbf{u}_i, \sigma_\xi^2)$$

$$\eta_{i,c} \mid \mathbf{v}_i, \bar{\eta}_{c,d}, \sigma_\eta^2 \sim \text{N}(\bar{\eta}_{c,d} + \mathbf{v}_i, \sigma_\eta^2)$$

Level 2 Priors on the Random Patient Effects $(\mathbf{u}_i, \mathbf{v}_i)$

$$\mathbf{u}_i, \mathbf{v}_i \mid \rho, \tau^2 \stackrel{iid}{\sim} \text{MVN}_2(\mathbf{0}_2, \Sigma_{\mathbf{u},\mathbf{v}})$$

where $\Sigma_{\mathbf{u},\mathbf{v}}$ has $\sigma_{\mathbf{u}}^2 = \sigma_{\mathbf{v}}^2 = \tau^2$ and covariances $\rho\tau^2$.

Priors of the Hierarchical Model

Level 2 Priors on the Mean Dose Effects $(\bar{\xi}, \bar{\eta})$

$$p(\bar{\xi}_{c,d} | \bar{\xi}_{c,-d}) \propto \phi(\bar{\xi}_{c,d} | \xi_{c,0}, \sigma_{\xi_{c,0}}^2) 1(\bar{\xi}_{c,d-1} < \bar{\xi}_{c,d} < \bar{\xi}_{c,d+1})$$

$$p(\bar{\eta}_{c,d} | \bar{\eta}_{c,-d}) \propto \phi(\bar{\eta}_{c,d} | \eta_{c,0}, \sigma_{\eta_{c,0}}^2) 1(\bar{\eta}_{c,d-1} < \bar{\eta}_{c,d} < \bar{\eta}_{c,d+1})$$

Level 2 prior means : $\xi_0 = (\xi_{1,0}, \xi_{2,0}), \quad \eta_0 = (\eta_{1,0}, \eta_{2,0})$

Level 2 prior variances : $\sigma_{\xi_0}^2 = (\sigma_{\xi_{1,0}}^2, \sigma_{\xi_{2,0}}^2), \quad \sigma_{\eta_0}^2 = (\sigma_{\eta_{1,0}}^2, \sigma_{\eta_{2,0}}^2)$

All fixed prior parameters : $\tilde{\theta}^{12 \times 1} = (\xi_0, \eta_0, \sigma_{\xi_0}^2, \sigma_{\eta_0}^2, \sigma_{\xi}^2, \sigma_{\eta}^2, \tau^2, \rho)$

Distributions of (ξ_i, η_i) and (Y, Z)

Latent variable distribution, after averaging over (u_i, v_i) :

$$\xi_i, \eta_i \mid d_i, \bar{\xi}, \bar{\eta}, \tilde{\theta} \stackrel{iid}{\sim} \text{MVN}_4 \left(\mu_{d_i}, \Sigma_{\xi, \eta} \right)$$

$\mu_{d_i} = (\bar{\xi}_{1, d_{i,1}}, \bar{\xi}_{2, d_{i,2}}, \bar{\eta}_{1, d_{i,1}}, \bar{\eta}_{2, d_{i,2}})$, depends on dose levels, but not numerical dose values. **Suppressing i , this induces**

$$p(\mathbf{y}, \mathbf{z} \mid \mathbf{d}, \boldsymbol{\theta}) = \Pr(Y_1 = y_1, Y_2 = y_2, Z_1 = z_1, Z_2 = z_2 \mid \mathbf{d}, \boldsymbol{\theta})$$

as a 4-dimensional integral of $\phi(\boldsymbol{\xi}, \boldsymbol{\eta} \mid \mu_{\mathbf{d}}, \Sigma_{\xi, \eta})$ **with mean dose effects $\boldsymbol{\theta} = (\bar{\boldsymbol{\xi}}, \bar{\boldsymbol{\eta}})$.**

Establishing a Prior

Obtain a non-informative prior $\pi(\theta | \tilde{\theta})$ by using prior expected sample size (ESS, Morita, et al., 2008) to calibrate $\tilde{\theta}$

Prior informativeness, quantified by ESS, is implied by $\tilde{\theta}$

Prototype Example: $Be(a, b)$ with mean $\mu = a/(a + b)$ and variance $\mu(1 - \mu)/(a + b + 1)$ has exact ESS= $a + b$.

More Complex Models: ESS is not at all obvious, so apply various algorithms to compute exact or approximate prior ESS

Prior Calibration Algorithm

1. Fix preliminary $\tilde{\theta} - (\xi_0, \eta_0) = (\sigma_{\xi_0}^2, \sigma_{\eta_0}^2, \sigma_{\xi}^2, \sigma_{\eta}^2, \tau^2, \rho)$.
2. Solve for (ξ_0, η_0) using the elicited prior probabilities for a set of doses and assumed hyperparameters.
3. Assume a vague, proper prior on θ with large $\sigma_{\xi_0}^2, \sigma_{\eta_0}^2$.
4. Simulate a large pseudo-sample of $\theta = (\bar{\xi}, \bar{\eta})$ values
5. Compute several probabilities of interest, $q(\theta)$. E.g. $q(\theta) = \Pr(Y_1 = 1 \mid d_1, \theta), \Pr(Y_1 = 2 \mid d_1, Y_1, Z_1 \mid \theta)$, etc.
6. Approximate the distribution of the simulated sample of each $q(\theta)$ with a $\text{Be}(a, b)$, match means and variances, and set $\text{ESS} = a + b$ for the prior of $q(\theta)$.
7. Calibrate $\tilde{\theta}$ by repeating Steps 1–6 until ESS values in the range 0.5 to 2.0 are obtained for all $q(\theta)$.

Objective Function

Utilities $U(y_c, z_c)$ of outcomes in each cycle $c = 1$ or 2 .

		Efficacy (Z_c)	
		No	Yes
Toxicity (Y_c)	No	35	100
	Yes	0	65

Mean utility of a_2 , given (d_1, Y_1, Z_1, θ) :

$$Q_2(a_2, d_1, Y_1, Z_1, \theta) = E\{U(Y_2, Z_2) \mid a_2, d_1, Y_1, Z_1, \theta\}$$

Cycle 2 Objective Function :

$$q_2(a_2, d_1, Y_1, Z_1, \mathcal{X}) = E\{Q_2(a_2, d_1, Y_1, Z_1, \theta) \mid a_2, d_1, Y_1, Z_1, \mathcal{X}\}.$$

Objective Function

Mean utility of $a_1 = d_1$ given θ , assuming $a_2^{opt}(d_1, Y_1, Z_1, \mathcal{X})$ will be used in cycle 2 :

$$Q_1(a_1, \theta) = Q_1(d_1, \theta) = E\{U(Y_1, Z_1) \mid d_1, \theta\}$$

Cycle 1 Objective Function : Given cycle 2 discount parameter $0 < \lambda < 1$, assume a_2^{opt} will be taken in cycle 2 :

$$q_1(d_1, \mathcal{X}) = E\{Q_1(d_1, \theta) \mid \mathcal{X}\} + \lambda E [E\{q_2(a_2^{opt}, d_1, Y_1, Z_1, \mathcal{X}) \mid \theta, d_1\} \mid d_1, \mathcal{X}]$$

The optimal cycle 1 action, a_1^{opt} , is the dose maximizing $q_1(d_1, \mathcal{X})$, **unless $q_1(d_1, \mathcal{X}) < U(0, 0)$, which implies $a_1^{opt} = NT$ and stops the trial.**

Additional Dose Acceptability Criteria

Motivation: We do not fully trust our model and decision-making scheme, and want actual oncologists to use this methodology.

1. In each cycle, do not skip untried dose levels when escalating.
2. Do not escalate in cycle 2 if toxicity was observed in cycle 1: $d_2 \leq d_1$ if $Y_1 = 1$, regardless of Z_1 .
3. d_1 is unacceptable if $q_1(d_1, \mathcal{X}) < U(0, 0)$
4. d_2 is unacceptable if $q_2(d_2, d_1, Y_1, Z_1, \mathcal{X}) < U(0, 0)$

$\mathcal{A}_1(\mathcal{X})$ = acceptable doses for cycle 1

$\mathcal{A}_{i,2}(d_{i,1}, Y_{i,1}, Z_{i,1}, \mathcal{X})$ = acceptable doses for cycle 2 of patient i

Adaptive Randomization

Motivation: Greedy adaptive algorithms that always take the optimal action risk getting stuck at a suboptimal action.

Define ϵ_i decreasing \downarrow in patient index i , with $\epsilon = (\epsilon_1, \dots, \epsilon_n)$.

Cycle 1 : The set of ϵ_i -optimal doses for cycle 1 is

$$\{d_1 \in \mathcal{A}_{i,1}(\mathcal{X}) : q_1(d_1, \mathcal{X}) > q_1(a_{1,i}^{opt}, \mathcal{X}) - \epsilon_i, \}.$$

In words: An ϵ_i -optimal dose has posterior mean payoff near the maximum (optimal) payoff.

Adaptive Randomization

Cycle 2 : The set of $(\epsilon_i/2)$ -optimal doses for cycle 2 given $(d_{i,1}, Y_{i,1}, Z_{i,1})$ is all acceptable d_2 having posterior mean utility within $\epsilon_i/2$ of the maximum.

We use $\epsilon_i/2$ because $q_2(a_2, d_1, Y_1, Z_1, \mathcal{X})$ is the posterior expected utility for cycle 2 only.

AR(ϵ)

Randomize the i^{th} patient among the ϵ_i -optimal doses in cycle 1 and $\epsilon_i/2$ -optimal doses in cycle 2.

The DTM2 Design: Trial Conduct

For the simulation study, we mimicked a typical phase I-II chemotherapy trial with 5 dose levels, but accounting for 2 cycles of therapy. Maximum sample size = 60, cohort size = 2.

Based on preliminary simulations, we set

$\epsilon_i = 20$ for the first 10 patients

$\epsilon_i = 15$ for the next 10 patients

$\epsilon_i = 10$ for the last 40 patients.

The DTM2 Design: Trial Conduct

1. Cohort 1 is treated at $d = 1$ in cycle 1, their (Y_1, Z_1) are observed, $\text{posterior}(\theta \mid \text{data})$ is computed, and actions taken for cycle 2. When (Y_2, Z_2) are observed from cycle 2, $\text{posterior}(\theta \mid \text{data})$ is updated.
2. Cohort 2 is enrolled after cohort 1 has been evaluated for cycle 1.
3. For cohorts 2, 3, ... , $q_1(d_1, \mathcal{X})$ is computed using $\lambda = 0.8$, \mathbf{a}^{opt} is identified, and $\text{AR}(\epsilon)$ is applied to choose \mathbf{a} .
4. Steps 1 – 3 are repeated until the trial is stopped early, or $N = 60$ is reached and a final optimal two-cycle \mathbf{a}_{sel} is chosen.

2-Cycle Comparators

We compared the DTM2 design to 2-cycle extensions of 3+3 algorithms and the continual reassessment method (CRM)

(3+3)a implicitly targets d with $P(Y_1 = 1 \mid d) \leq 0.17$

(3+3)b implicitly targets d with $P(Y_1 = 1 \mid d) \leq 0.33$

The extended (3+3) methods both choose d_2 as follows:

If $Y_1 = 1$, then $d_2 = d_1 - 1$. (Tox in cycle 1 \Rightarrow de-escalate)

If $Y_1 = 0$, then $d_2 = d_1$. (No Tox in cycle 1 \Rightarrow repeat d_1)

2-Cycle Comparators

CRM model $\Pr(Y_1 = 1|d) = p_d^{\exp(\alpha)}$, with $\alpha \sim N(0, 2)$ choosing $p_1 < \dots < p_5$ using “getprior” in “dfcrm” (Cheung, 2011)

Cycle 1 : Each patient's d_1 was chosen

(i) to have posterior mean $\Pr(\text{Tox})$ closest to 0.30

(ii) imposing the “do not skip an untried dose” rule.

Cycle 2 : d_2 was chosen using the same rule as for the extended (3+3) methods. Also, d_2 was unacceptable if

$$\Pr\{\Pr(Y_1 = 1 \text{ or } Y_2 = 1) > 0.50 | \mathcal{X}, \mathbf{d}\} > 0.90.$$

In words: A cycle 2 dose is too high if it makes it likely that the probability of at least one toxicity in two cycles is over 50%.

Simulation Study: Evaluation Criteria

Empirical mean total utility for n patients in the trial :

$$\bar{U} = \sum_{i=1}^n \{U(Y_{i,1}, Z_{i,1}) + U(Y_{i,2}, Z_{i,2})\} / n$$

setting $U(Y_{i,2}, Z_{i,2}) = U(0, 0)$ if $a_{i,2} = NT$.

Empirical mean total payoff for all patents :

$$\bar{\bar{U}} = \frac{1}{N} \sum_{r=1}^N \bar{U}^{(r)}.$$

indexing simulated trials by $r = 1, \dots, N$. This quantifies ethical desirability, given $U(y, z)$.

Simulation Study: Evaluation Criteria

Expected payoff of $a_{1,sel}$, given θ^{true} :

$$Q_{1,sel}(a_{1,sel}) = E\{U(Y_1, Z_1) \mid a_{1,sel}, \theta^{true}\},$$

Expected payoff of $a_{2,sel}$:

$$Q_{2,sel}(a_{2,sel}) = E\{U(Y_2, Z_2) \mid a_{1,sel}, a_{2,sel}(y_1, z_1), y_1, z_1, \theta^{true}\}$$

where $E\{U(Y_2, Z_2) \mid a_{1,sel}, a_{2,sel}(y_1, z_1), y_1, z_1, \theta^{true}\} = U(0, 0)$ if $a_{2,sel}(y_1, z_1) = NT$.

Expected payoff to a future patient treated using a_{select} :

$$Q_{sel}(a_{sel}) = Q_{1,sel}(a_{1,sel}) + \lambda Q_{2,sel}(a_{2,sel}).$$

Simulation Study: Evaluation Criteria

Denote $\delta_{i,2} = 1$ if patient i received $a_{i,2} = d_2$, and $\delta_{i,2} = 0$ if $a_{i,2} = NT$.

Empirical Toxicity and Efficacy Probabilities

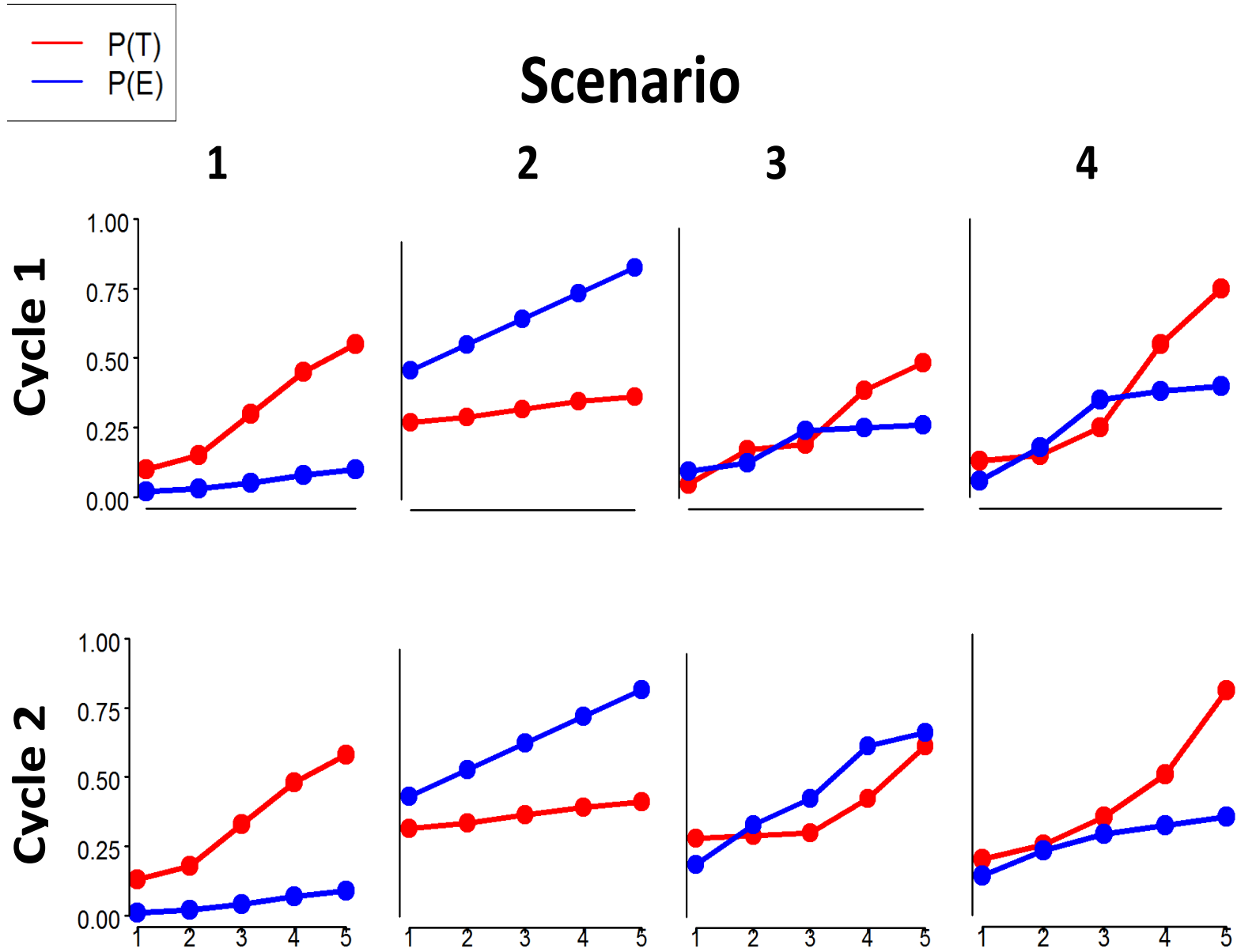
$$\Pr(\text{Tox}) = \frac{1}{n} \sum_{i=1}^n \frac{1(Y_{i,1}=1) + \delta_{i,2}1(Y_{i,2}=1)}{1 + \delta_{i,2}}$$

$$\Pr(\text{Eff}) = n^{-1} \sum_{i=1}^n \frac{1(Z_{i,1}=1) + \delta_{i,2}1(Z_{i,2}=1)}{1 + \delta_{i,2}}$$

Simulation Scenarios

True $(p_T, p_E)^{true}$ under the four simulations scenarios.

Scenario	Cycles	Doses				
		1	2	3	4	5
1	1	(0.10, 0.02)	(0.15, 0.03)	(0.30, 0.05)	(0.45, 0.08)	(0.55, 0.10)
	2	(0.13, 0.01)	(0.18, 0.02)	(0.33, 0.04)	(0.48, 0.07)	(0.58, 0.09)
2	1	(0.30, 0.50)	(0.32, 0.60)	(0.35, 0.70)	(0.38, 0.80)	(0.40, 0.90)
	2	(0.33, 0.45)	(0.35, 0.55)	(0.38, 0.65)	(0.41, 0.75)	(0.43, 0.85)
3	1	(0.05, 0.10)	(0.18, 0.13)	(0.20, 0.25)	(0.40, 0.26)	(0.50, 0.27)
	2	(0.30, 0.20)	(0.31, 0.35)	(0.32, 0.45)	(0.45, 0.65)	(0.65, 0.70)
4	1	(0.13, 0.06)	(0.15, 0.18)	(0.25, 0.35)	(0.55, 0.38)	(0.75, 0.40)
	2	(0.20, 0.14)	(0.25, 0.23)	(0.35, 0.29)	(0.50, 0.32)	(0.80, 0.35)



Scenario Descriptions

Scenario 1: $d = 1, 2, 3$ **safe**, $d = 3, 4$ **too toxic**, but all doses have low efficacy
 \Rightarrow **The optimal action is to stop the trial early.**

Scenario 2: $0.30 \leq p_T(d_1) \leq 0.40$ and $.33 \leq p_T(d_2) \leq 0.43$ with very high
 $p_E(d_1)$ and $p_E(d_2) \Rightarrow$ **Big payoff for escalating to higher doses.**

Scenario 3: $a_1^{opt} = 3$, with $a_2^{opt}(Y_1 = 0) = 4$ and $a_2^{opt}(Y_1 = 1) = 2 \Rightarrow$ **Accounting for Y_1 is very important when choosing d_2**

Scenario 4: **The optimal doses happen to coincide with what the 3+3 and CRM choose while ignoring efficacy. “A stopped clock is right twice a day.”**

Optimal Actions Under the 4 Scenarios

Scenario	a_1^{opt}	a_2^{opt}			
		(0,0)	(0, 1)	(1,0)	(1,1)
1	<i>NT</i>	<i>NT</i>	<i>NT</i>	<i>NT</i>	<i>NT</i>
2	5	5	5	4	4
3	3	4	4	2	2
4	3	3	3	<i>NT</i>	2

Percent Completed Trials

Scenarios	DTM2	(3+3)a	(3+3)b	Extended CRM
1	2.3	88.6	96.5	99.8
2	99.4	39.2	64.7	93.1
3	79.4	99.6	99.2	99.8
4	96.7	83.2	94.7	99.8

In Scenario 1, DTM2 finds that all doses are inefficacious, and stops the trial 97.7% of the time. The other 3 methods all ignore efficacy and continue to treat patients.

Summary of Simulation Results

Scenario		DTM2	(3+3)a	(3+3)b	Extended CRM
2	\bar{U}	136.35	123.32	117.78	116.38
	Q_{select}	135.76	103.85	104.48	103.17
	Pr(Tox)	0.39	0.30	0.34	0.37
	Pr(Eff)	0.72	0.57	0.55	0.57
3	\bar{U}	94.23	85.72	85.49	89.29
	Q_{select}	84.39	77.97	80.13	78.29
	Pr(Tox)	0.38	0.27	0.28	0.30
	Pr(Eff)	0.38	0.26	0.27	0.33
4	\bar{U}	75.84	81.83	79.85	84.88
	Q_{select}	69.49	74.92	75.76	79.12
	Pr(Tox)	0.51	0.25	0.27	0.29
	Pr(Eff)	0.29	0.22	0.21	0.29

Some Conclusions

- 1: A “Bayes Meets Bellman” dose-finding method, adaptive between and within patients, with lots of model-method-computing machinery.**
- 2: Very good properties compared to “toxicity only” 2-cycle methods – but not in all cases.**
- 3: Are 2 cycles better than 1 ? Sometimes Yes, sometimes No.**
- 4: Very sensitive to $U(y, z) \Rightarrow$ It is essential to calibrate $U(y, z)$ with the physician/scientists when designing an actual trial.**
- 5: $U(y, z)$ is not enough! Additional safety/efficacy restrictions are needed.**