General Methods for Power Calculation

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General Approach to Power Analysis

- Power: An introduction
- Technical details
- General power methods
- 3 Examples
- Conclusion

What is statistical power?

"The power of a statistical test is the probability that it will yield statistically significant results."

Jacob Cohen

Power availability

- Programs widely available
- SAS: PROC POWER, GLMPOWER: Packages
- PASS,G*Power: Standalone programs
- R methods
- Multiple websites online

Power methods lag behind methods development General approaches are needed which are less specific

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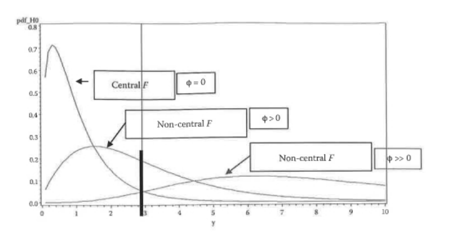
The non-centrality parameter φ

General concept

- Central F, H_0 true
- $\varphi = 0$
- Non-central F, H_0 false
- Deviation from H_0
- $\varphi > 0$

Distributional view



Stroup, 2013

$$\varphi = (\mathbf{K}'\beta - \psi)' \left[\mathbf{K}' (\mathbf{X}'\mathbf{W}\mathbf{X})^{-} \mathbf{K}\right] (\mathbf{K}'\beta - \psi)$$
$$= F \times \text{rank}(K) = F \times \text{df}_n$$

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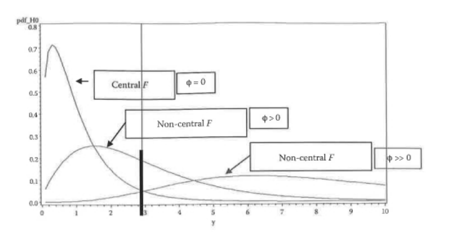

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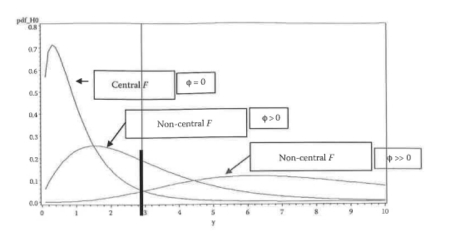

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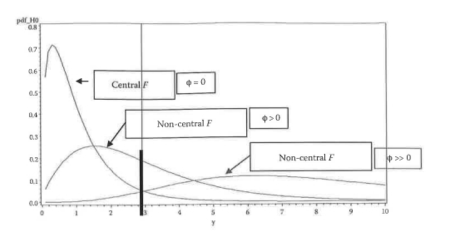

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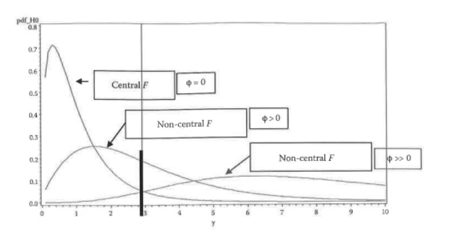

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Using the non-centrality parameter

With non-centrality parameter, power is easy to calculate

Power is the "proportion under non-central F distribution that exceeds the critical value for the null hypothesis"

$$\text{Power} = 1 - \text{Fprob}(F, \text{df}_n, \text{df}_d, \omega) \tag{1}$$

This is a non-central F calculator, widely available

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Obtaining the appropriate F value

- Obtained from analysis tool
- General analysis tools are available
- GLIMMIX in SAS
    - Standard LM
    - Generalized LM
    - Multi-level models
- Muller, O'Brien, Stroup: GLM
- Stroup approach: Parameters in program
- Create exemplar dataset

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- Defined by problem
- Means and variances
- Covariances: Define RM models
- More difficult: Linear growth model

Key: Exemplar dataset is not random dataset
Structure defined by power problem
Structure reflects actual behavior

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General method for power analysis: GLIMMIX-Power

The general approach:

- Define exemplar dataset
- Use appropriate computational method
- Compute $\varphi = F * rank(\mathbf{K})$
- Compute $F_c = \text{Finv}(\text{df}_n, \text{df}_d, .05)$
- Compute power $= 1 - \text{Fprob}(F_c, \text{df}_n, \text{df}_d, \varphi)$

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General method for sample size determination

The general approach:

- Compute $\varphi = F * rank(\mathbf{K})$
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Search: Simple grid search

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Alternative: Simulation

Simulation is often used

Determine empirical rejection rate

Problems with simulation

- Simulation datasets are NOT correct
- Long-run behavior:
  - Simulated parameter ests = model-defined
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  - Assumes symmetry in φ
- Not clear how well simulation works

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Two sample test: Basic model

Two sample test setup

$$\mathbf{Y} = \mathbf{X}\beta + \epsilon \quad (2)$$

$$\mathbf{Y} \sim N(\mu, \sigma^2\mathbf{I}) \quad (3)$$

$$\text{Test statistic:} \quad \mathbf{c}\beta - \theta \quad (4)$$

Power: Well-defined closed-form

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Two sample test: Exemplar dataset

Exemplar dataset construction process:

- Generate random dataset
- Group i: Impose μ_i and σ_i^2, n_i obs
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- Compute power and/or sample size as above

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Two sample test: Comparisons

Three power methods compared

- Closed form (`PROC POWER`)
- `GLIMMIX`-Power
- Simulation: ES: (.25 .5 1), n_i: (10,20,30,50,75,100), 2000 reps, examine subsets

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Two sample test: Results

				Simulation, q3, Power		
n1,n2	Power	Diff	N-GLIM	100	250	500
10	0.083	0	457	0.025	0.016	0.012
20	0.120	0	482	0.040	0.022	0.015
30	0.159	0	490	0.040	0.024	0.021
50	0.236	0	497	0.046	0.036	0.025
75	0.331	0	500	0.051	0.031	0.027
100	0.438	0	502	0.065	0.042	0.038

Diff: Compares `POWER` and `GLIMMIX`-based power

Simulation values can be discrepant, esp in small reps

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75	0.331	0	500	0.051	0.031	0.027
100	0.438	0	502	0.065	0.042	0.038

Diff: Compares `POWER` and `GLIMMIX`-based power
Simulation values can be discrepant, esp in small reps

Multilevel RM: Basic model

Multilevel RM setup

$$\mathbf{Y} = \mathbf{X}\beta\epsilon \quad (5)$$

$$\mathbf{Y} \sim N(\mu, \mathbf{\Sigma}) \quad (6)$$

$$\text{Test statistic:} \quad \mathbf{c}\beta\mathbf{u} = \theta \quad (7)$$

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pause Less common situation

Power: No well-defined approach

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Exemplar dataset construction process:

- Generate random dataset, n_i obs
- Orthogonalize each group, impose μ_{it} and Σ structure
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Three power methods compared

- Closed form (`PROC GLMPOWER`) (multivariate)
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Multilevel RM: Results

		Power Est		abs(Diff) - Sim	
den df	F	gp	SAS	vs gp	vs SAS
36	2.19	0.419	0.268	0.041	0.172
36	3.16	0.569	0.677	0.035	0.103
36	1.67	0.328	0.366	0.036	0.052
76	1.58	0.325	0.225	0.031	0.127
76	2.31	0.456	0.333	0.046	0.135
76	1.63	0.335	0.262	0.033	0.102

GLIMMIX-based power better than PROC GLMPOWER

Many other situations, GLIMMIX-based power better

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$$Y_{gij} = a_{gi} + b_{gi}x_{gij} + e_{gij} \quad (9)$$

$$\text{RGCM} : Y_{ij} = a_i + b_i x_{ij} + e_{ij} \quad (10)$$

$$\text{RM} : Y_{ij} = F_{ij} + e_{ij} \quad (11)$$

Relationship: (means are obvious)

$$\sigma_t^2 = \sigma_a^2 + t^2\sigma_b^2 + \sigma_e^2 \quad (12)$$

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Same setup, different sample sizes

Test	n	Power	abs(diff)	S Size
Grp:V=6	10	0.145	0.039	194
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2. Values consistent with other approaches
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